Name:

Mulberry Academy Shoreditch

Sixth Form

Summer Assignment

[Core Maths]

A6.1 AER and compound interest

Before you start

- You should already know how to increase an amount by a given percentage.
- You will need to be able to use your calculator to find the nth root of a number.

Why do this?

Compound interest and the annual equivalent rate (AER) play an important role in everyday investments, especially those taking place over more than two or three years.

Objectives

- Be able to calculate the final amount and the interest on an investment.
- Be able to calculate the annual equivalent rate (AER) of an investment.

Get Ready

1 £6000 is invested at 4% p.a. Work out the value of the investment after one year.

2 Use a calculator to work out:

a 2^{10} b $6000 \times \left(1 + \frac{4}{100}\right)^5$

3 Use a calculator to work out $729^{\frac{1}{6}}$

Key Points

- Compound interest is interest paid on the amount and the interest already earned.

Example 1

Katie invests £3000 at 3.4% compound interest.
Work out the value of her investment after 2 years.

$$\text{Interest after 1 year} = \frac{3000 \times 3.4}{100} = £102$$

$$\text{Value after 1 year} = 3000 + 102 = £3102$$

Value after 1 year = investment + interest after 1 year

$$\text{Interest in year 2} = \frac{3102 \times 3.4}{100} = £105.47$$

$$\text{Value after 2 years} = 3102 + 105.47 = £3207.47$$

Value after 2 years = value after 1 year + interest in year 2

Exercise 6A

C

1 Jim invests £2000 at 3% p.a. compound interest for 2 years. Work out the final amount.

2 Jade invests £1500 at 3% compound interest for 10 years. Work out the final amount.

Key Points

- Compound interest can also be calculated using a formula.
- When £P is invested in an account paying r% compound interest per annum (p. a.), the value, £V, of the investment after n years is given by:

$$V = P\left(1 + \frac{r}{100}\right)^n$$

- When £P is invested in an account for n years to produce an investment of value £V, the annual equivalent rate of interest (AER) is given by:

$$\alpha = 100\left(\left(\frac{V}{P}\right)^{\frac{1}{n}} - 1\right) \text{ where } \left(\frac{V}{P}\right)^{\frac{1}{n}} = \sqrt[n]{\left(\frac{V}{P}\right)}$$

Example 2

Katie invests £3000 at 3.4% compound interest.
Work out the value of her investment after 5 years.

$$V = 3000 \times \left(1 + \frac{3.4}{100}\right)^5 = 3000 \times 1.034^5 = £3545.88$$

Substitute $P = 3000$, $r = 3.4$ and $n = 5$ into the compound interest formula.

Example 3

Josh invested £5000 in an account.
After 5 years the value of the account was £7000.
Work out the annual equivalent rate (AER) of the account.

$$\alpha = 100 \times \left(\left(\frac{7000}{5000}\right)^{\frac{1}{5}} - 1\right) = 100 \times \left(1.4^{\frac{1}{5}} - 1\right) = 6.96\%$$

Substitute $P = 5000$, $V = 7000$, and $n = 5$ into the formula

Example 4

Adam invested some money into an account which paid interest annually.
In the first year the account paid 2% compound interest.
In the second year the account paid 4% interest,
and in the third year the account paid 6% interest.
Work out the annual rate of interest (AER) of the account.

$$V = P\left(1 + \frac{2}{100}\right)\left(1 + \frac{4}{100}\right)\left(1 + \frac{6}{100}\right) = 1.124\,448P$$

Use the compound interest formula for 3 successive years with the correct value of r each time

Use the AER formula with $n = 3$ and $V = 1.124\,448P$

$$\text{AER} = 100 \times \left(\left(\frac{1.124448P}{P}\right)^{\frac{1}{3}} - 1\right) = 100 \times \left(1.124448^{\frac{1}{3}} - 1\right) = 3.99\%$$

NB As a way of checking, Adam's investment should give the same return as if he had invested in an account paying 3.99% p.a. compound interest for 3 years

$$P \times \left(1 + \frac{3.99}{100}\right)^3 = 1.1245\ldots \times P$$

which compares well with the $1.124448P$ above, the difference being due to the rounding of the AER to 2 decimal places.

Example 5

Holly invests £10000 in an account with an annual equivalent rate AER of 5%. She gets the interest paid half yearly. Work out the value of her first half yearly interest payment.
Suppose her half yearly interest payment rate is x%, then:

$$\left(1 + \frac{x}{100}\right)^2 = \left(1 + \frac{5}{100}\right)$$

This line comes from using compound interest for two successive half years and setting it equal to using 5% for 1 year. This line will be true no matter how much money is invested.

$$\text{So: } 1 + \frac{x}{100} = \sqrt{1.05} \quad x = 100 \times (\sqrt{1.05} - 1) = 2.469\,5\,\%$$

The amount of money added to the account is £10 000 × 2.469 5 % = £246.95.

Exercise 6B

B

1 Work out the value of these investments in accounts paying annual compound interest after the number of years stated.

	Initial Investment	Annual Interest rate	Number of years
a	£5000	5%	3
b	£2000	4%	5
c	£500	3.5%	6
d	£250	2.8%	10
e	£750	4.7%	18

2 Bill invests £5000 in an account paying 4% compound interest p.a. for 6 years. Work out the total interest that the account earns. AO3

3 Mr Smith invests £10 000 in a savings scheme for 6 years. The AER of the savings scheme is 3.2%. Mr Smith will have to pay tax at 40% on the total interest he gets at the end of the 6 years. Work out how much tax Mr Smith will have to pay on the investment. AO3

4 Every year Jim invests £1000 in an account paying 3% compound interest p. a. Work out the amount of money in the account at the end of the third year. AO2

5 Mrs Newton wants to invest some money to pay for her son to attend university. She plans to invest in an account which pays 4.8% per annum compound interest. How much will she have to invest so that the account is worth £6000 after 5 years? AO3

6 Ravi has £8000 to invest. He intends to leave it in his account for 6 years. What rate, per annum, of compound interest will enable the value of the account to reach £10 000 after 6 years? AO3

7 An account pays 6% compound interest per annum. How many years will the investment have to be in place before its value doubles? AO3

8 Work out the annual equivalent rate (AER) for each of these investments. AO3

	Initial Investment	Number of years (n)	Value of the investment after n years
a	£5000	3	£6000
b	£2000	5	£2200
c	£500	6	£720
d	£250	10	£318
e	£750	18	£1710.50

A

9 James invests £1000 in an account. For the first year the account paid interest at 5% p.a. For the second year the account paid interest at 3.5% p.a. Work out the annual equivalent rate (AER) of interest on this account. Give your answer correct to 4 significant figures. AO2

10 Annette invested £2500 in an account. In the first year the interest rate was 3%, in the second year 5% and in the third year 7%. AO3

a Work out the value of Annette's account at the end of 3 years.

b Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

A AO3

11 Naseem invests £20 000 in an account. For the first two years the account pays 4% per annum compound interest, and for the next three years the account pays 6% per annum compound interest.

a Work out the value of the account after 5 years.

b Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

AO2

12 A savings plan lasts for 5 years. For the first year the interest rate is 2%. The interest rate increases by 1% every year for the life of the savings plan.
Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

AO3

13 An account pays 4% compound interest on the amount in the account every six months. What is the annual equivalent rate of interest?

A6.2 Cost of living index

Before you start

- You should be able to calculate with money.
- You should be able to calculate a percentage of an amount.

Why do this?

Basic money calculations are essential in modern life and having an understanding of the cost of living is useful when judging the value of wage rises.

Objectives

- Gain an understanding of financial mathematics.
- Be able to calculate wage increases which are in line with cost of living increases.

Get Ready

1 Work out 3% of £180.
2 Increase £320 by 5%.

Key Points

- The cost of living index is a measure of how prices increase. It is linked with the idea of inflation of prices.
- The cost of living index has a base year when the index is set equal to 100.
- The cost of living index increases by an amount each year, which depends on the costs of a typical set of items that people buy.

Example 1

Jim pays rent on a flat. Each year the rent increases in line with the cost of living index.

In 2010 the rent was £420 per month and the cost of living index was 100
In 2011 the cost of living index was 103.5
Work out what Jim's rent will be in 2011.

The cost of living increases by 3.5%.

The increase in the cost of living is 103.5 − 100 out of 100.

Jim's rent will increase by $3.5\% = 420 \times \frac{3.5}{100} = 14.7$.
Jim's new rent will be £434.70 per month.

Example 2 Here is Mr Lincoln's bank statement from 1 April to 28 April. Some items are missing.

A deposit in an account happens when money is added to the account.

Date	Deposit (£)	Withdrawal (£)	Balance (£)
1.4.2012			3420.26
6.4.2012		200.00	3220.26
13.4.2012	312.51		3532.77
20.4.2012		250.00	.....
28.4.2012	1250.00		.....

The balance is the amount of money in the account.

a Write down how much was in the account on 1 April.
b Copy and complete Mr Lincoln's bank account.
c Mr Lincoln wants to know whether he can afford to pay a deposit of £4500 on a car. Can he afford it?

a £3420.26
b Missing items are £3282.77 and £4532.77
c Yes as the balance of his account is more than £4500.

Exercise 6C

F AO3

1 John earns £250. He gets a wage rise of 10%. Work out his new wage.

2 Ben can buy 4 tins of tomatoes at 59p each or he can buy a bargain pack of 4 tins of tomatoes for £1.99. Work out how much he can save.

3 A litre of fuel costs 121.9p.
a Lizzie buys 25 litres of fuel. How much will she have to pay?
b Amir buys £40 worth of fuel. How much fuel does he buy?

E AO3

4 Annie's rent is £112 per week. She gets a 10% reduction. Work out her new rent.

5 A student railcard costs £26. The railcard allows a student to buy rail tickets with $\frac{1}{3}$ off the normal price. Anya wants to get a rail ticket. The normal price is £114.
How much money can she save by buying a railcard and using it to reduce the price of the rail ticket?

E AO2

6 a Lethna has £1.80. She wants to buy a drink and fries.
What are the different combinations that can she buy?

b Ken buys:
2 double burgers with cheese, 1 large portion fries and 1 large cola.
He pays with a £10 note. He gets the best price.
What change should he get?

Ben's Burger Bar

Burgers

Single burger	£0.85
Single burger with cheese	£0.95
Double burger	£1.55
Double burger with cheese	£1.70

Fries		Cola	
Regular	£0.65	Regular	£0.85
Large	£0.99	Large	£1.10

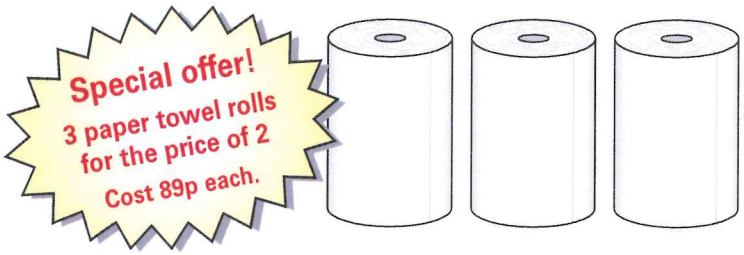
Meal Deals

Regular
Single burger with cheese, regular fries and regular cola £2.09

Large
Double burger with cheese, large fries and large cola £3.49

AO3

7



Natasha wants to buy 6 paper towel rolls. Work out how much she can save by using the special offer.

8 Javier gets the bus to and from work each day. He can get a daily return costing £2.90 or he can get a 5-day return costing £12.
How much will he save each week by buying a 5-day return?

9 Fred can buy a season ticket to watch his football team's home games. It will cost him £720 and allows him to attend all his team's home games.
Without a season ticket it will cost Fred £32 to attend each home game.
Fred's football team plays 23 home games.
Work out how much Fred would save by buying a season ticket.

D

10 Saeed earns £18 000 in a year. He does not pay tax on the first £6000 of the £18 000.
He pays tax 20% on the remainder.
Work out how much tax Saeed has to pay.

AO3

11 In 2009 Jenna found she had spent £3000 on rent, £800 on heating and £400 on rates.
In 2010, her rent for the year increased by 5%, heating by 15% and rates by 10%.
Work out the total increase in the amount of money that Jenna spent on these three items in 2010.

12 Oscar buys a car. The cash price of the car is £25 000.
Oscar pays a deposit of 30% of the cash price, followed by 24 monthly payments of £800 each.
How much altogether does Oscar pay for the car?

13 On average the cost of living is 5% higher in Cambridge than in Swindon.
Sophie spends £25 000 each year living in Swindon. How much would it cost her to live in Cambridge?

C

* **14** Jodie buys a car. The cash price of the car is £24 000. AO2
Jodie pays a deposit of 25%, followed by 24 monthly payments of £900 each.
Bob says that overall Jodie will be paying more than 120% of the cash price of the car.
Is Bob correct? Explain your answer.

15 The cost of living index was 100 in 2005. It increased by 3% by the start of 2006. AO3
Leonie gets a pay rise at the start of 2006 in line with inflation.
In 2005 she earned £1400 per month.
How much would she earn each month after her pay rise?

16 The cost of living index was 100 in 2005. It increased to 110.8 in 2009.
The living costs of Steve's family increased in line with inflation. In 2005 that cost was £800 per week.
How much was it in 2009?

* **17** The cost of living index was 100 in 2005. It increased to 108.5 in 2008. AO2
The cost of a litre of petrol in 2005 was 88p. The cost of a litre of petrol in 2008 was £1.03.
Did the cost of petrol go up by a bigger percentage than the cost of living?
Explain your answer.

18 The cost of living index was 100 in 2005. It increased to 114 in 2010. The national minimum adult wage in 2005 was £5.05 per hour. AO3

a What would the national minimum adult wage have to be in 2010 to keep pace with inflation?
The national minimum wage for 16-17 year olds in 2005 was £3.00 per hour. In 2010 it was £3.57.

*b Has the national minimum wage for 16-17 year olds kept pace with inflation?
Explain your answer.

* **19** In 2007, Fran earned £20 000 per year. She spent 15% of her earnings on rent. AO2 AO3
By 2009, Fran's wage had increased by 5%. Her rent was now £3500 per year.
Does Fran spend a greater or smaller percentage of her earnings on rent in 2009 than she did in 2007?
You must give a reason for your answer.

20 Rail operators are allowed to raise fares by the cost of living index increase + 1%. In 2010, the cost of living increase was 4.5%.
The fare from Bristol to London in 2010 was £120.
What is the new fare in 2011 assuming the rail operator applies the maximum increase?

Review

- When £P is invested in an account paying r% compound interest per annum (p. a.), the value, £V, of investment after n years is given by:
$V = P\left(1 + \frac{r}{100}\right)^n$
- When £P is invested in an account for n years to produce an investment of value £V, the annual equivalent rate of interest (AER) is given by:
$\alpha = 100\left(\left(\frac{V}{P}\right)^{\frac{1}{n}} - 1\right)$ where: $\left(\frac{V}{P}\right)^{\frac{1}{n}} = \sqrt[n]{\left(\frac{V}{P}\right)}$
- The cost of living index gives information about the increase in cost of a set of typical items for a family over one year.

Answers

Chapter 6

A6.1 Get Ready answers

1 £6240
2 **a** 1024 **b** 7299.917
3 3

Exercise 6A

1 £2121.80
2 £2015.87

Exercise 6B

1 **a** £5788.13 **b** £2433.31 **c** £614.63
d £329.51 **e** £1714.36
2 $5000 \times 1.04^6 = £\,6326.60$ Interest $= £1326.60$
3 $10\,000 \times 1.032^6 - 10\,000 = £2080.31$
Tax $= £832.12$
4 $1000 \times 1.03^3 + 1000 \times 1.03^2 + 1000 \times 1.03 = £3183.63$
5 $P \times \left(1 + \frac{4.8}{100}\right)^5 = 6000 \quad P = \frac{6000}{1.048^5} = £4746.19$
6 $r = 100 \times \left(\left(\frac{10000}{8000}\right)^{\frac{1}{6}} - 1\right) = 3.79\%$
7 $P \times 1.06^n = P \times 2$
T&I gives $n = 11.9$ so after 12 full years.
8 **a** $100 \times \left(\left(\frac{6000}{5000}\right)^{\frac{1}{3}} - 1\right) = 6.27\%$
b $100 \times \left(\left(\frac{2200}{2000}\right)^{\frac{1}{5}} - 1\right) = 1.92\%$
c $100 \times \left(1.44^{\frac{1}{6}} - 1\right) = 6.27\%$
d $100 \times \left(1.272^{\frac{1}{10}} - 1\right) = 2.44$
e $100 \times \left(\left(\frac{1710.50}{750}\right)^{\frac{1}{18}} - 1\right) = 4.69\%$
9 $V = 1000 \times 1.05 \times 1.035 = £1086.75$
$\alpha = 100 \times \left(\left(\frac{1086.75}{1000}\right)^{\frac{1}{2}} - 1\right) = 4.247\%$
10 **a** $2500 \times 1.03 \times 1.05 \times 1.07 = £2893.01$
b $100 \times \left(\left(\frac{2893.10}{2500}\right)^{\frac{1}{3}} - 1\right) = 4.988\%$
11 **a** $20\,000 \times 1.04^2 \times 1.06^3 = £25\,764.06$
b $100 \times \left(\left(\frac{25\,764.06}{20\,000}\right)^{\frac{1}{5}} - 1\right) = 5.195\%$
12 $V = P \times 1.02 \times 1.03 \times 1.04 \times 1.05 \times 1.06$
$\alpha = 100 \times \left(\left(\frac{1.02 \times 1.03 \times 1.04 \times 1.05 \times 1.06}{1}\right)^{\frac{1}{5}} - 1\right)$
$= 3.990\%$
13 $\alpha = 100 \times (1.04^2 - 1) = 8.16\%$

A6.2 Get Ready answers

1 £5.40
2 £336

Exercise 6C

1 £275
2 236p − 199p = 37p
3 **a** £30.47 or £30.48 **b** 32.81 litres
4 £100.80
5 $\frac{114}{3} = 38$ so £12
6 **a** Regular fries with regular cola, Regular fries with large cola
b £4.81
7 £1.78
8 £2.50
9 £16
10 £2400
11 £150 + £120 + £40 = £310
12 £7500 + £19 200 = £26 700
13 £26 250
14 No. £6000 + £21 600 = £27 600, 1.20 × £24 000 = £28 800
15 £1442
16 £886.40 per week
17 88p × 1.085 = 95.48p which is less than 103p so the price of petrol has risen faster
18 **a** £5.76
b 300p × 1.14 = 342p so is above inflation.
19 New wage = £21 000.
New percentage $= \frac{3500}{21\,000} \times 100 = 16.7\%$, which is greater than 2007.
20 £126.60

A as a percentage of B

1) 36 marks in a test worth 60 marks
2) £2.50 out of £12
3) 40 cm out of 3.5 m
4) 15 minutes out of 2 and a half hours
5) 40p out of £8
6) 70g out of 950g

Percentage increase or decrease

(Hint: Think about percentage profit or loss. Check the formula from last lesson)

1) Fred buys a car for £3,000. He sells it 2 years later for £2,200. Does he make a profit or a loss? What is this as a percentage of the original amount?

2) Mia spends £8.50 on sweets and decorations. She makes gift bags and sells them for £1 each. If Mia sells 15 bags does she make a profit or a loss? What is this as a percentage of the original amount she spent?

3) A restaurant buys 3 cases of drinks for £30. There are 12 bottles in each case. They sell all the bottles for £3.00 each. Do they make a profit or loss? What is this as a percentage of the original amount the spent?

4) Roger buys a box of 12 doughnuts for £8. He sells them for 90p each. If he sells them all does he make a profit or a loss? What is this as a percentage of the original amount he spent?

5) Janet buys a TV for £600. She sells it 3 years later for £250. Does she make a profit or a loss? What is this as a percentage of the original amount?

Percentage of a quantity

(Hint: multipliers)

1) a) 20% of £300 b) 16% of £145 c) 90% of 2.5m d) 84% of 60cm

2) 125 people work at a hotel, 12% work in the kitchen. How many people work in the kitchen?

3) 27 pupils in a class, 21 are girls. What percentage are boys?

4) A sweet shop sells 16 different types of gift boxes. Only 20% of these contain chocolate. How many contain chocolate?

5) I invest £260 in a savings account that pays 6% interest. How much do I have after 1 year?

Finding the new amount after a percentage increase or decrease

(Hint: use increase/decrease multipliers)

1) a) Increase £130 by 15% b) Increase 2.6m by 45%
c) Decrease £300 by 16% d) Decrease 7m by 34%

2) A Shop has a sale and decreases its prices by 35%. Before the sale a TV cost £450, how much is it now?

3) Anita receives a pay increase of 4%. She was earning £7.50 per hour, now how much is she earning?

4) A catalogue is selling clothes at a 12% discount of prices in the high street store. How much would a Jumper be in the catalogue if it was £55 in store?

5) Prices of chocolate increase by 8% because of a new tax. A large chocolate bar used to cost 75p, how much is it now?

Compound interest

(Hint: Think about how many times you multiply by the increase/decrease)

1) An antique vase is worth £300 when I buy it, its value increases by 2% each year. How much will it be worth in 8 years time?

2) A coffee machine was bought for £2,000, it depreciates in value by 8% each year. How much is it worth after 3 years?

3) Jenny invests £630 in a bank account with a compound interest rate of 4%. How much will she have in 6 years time?

4) Kenny invests £1,550 in an account with 3% compound interest. How much will he have in 3 years?

5) Raj invests £20,000 in a savings account at a compound interest rate of 4% per year. How long until he doubles his money?

Reverse percentage

(Hint: Do the opposite of find a new amount after a % increase/decrease)

1) The price of a new dishwasher is £376. This price includes VAT at a rate of 17.5%. What was the price before VAT was added?

2) A music shop has a sale, all item are reduced by 30%. The sale price of a CD is £3.36. What was the original price?

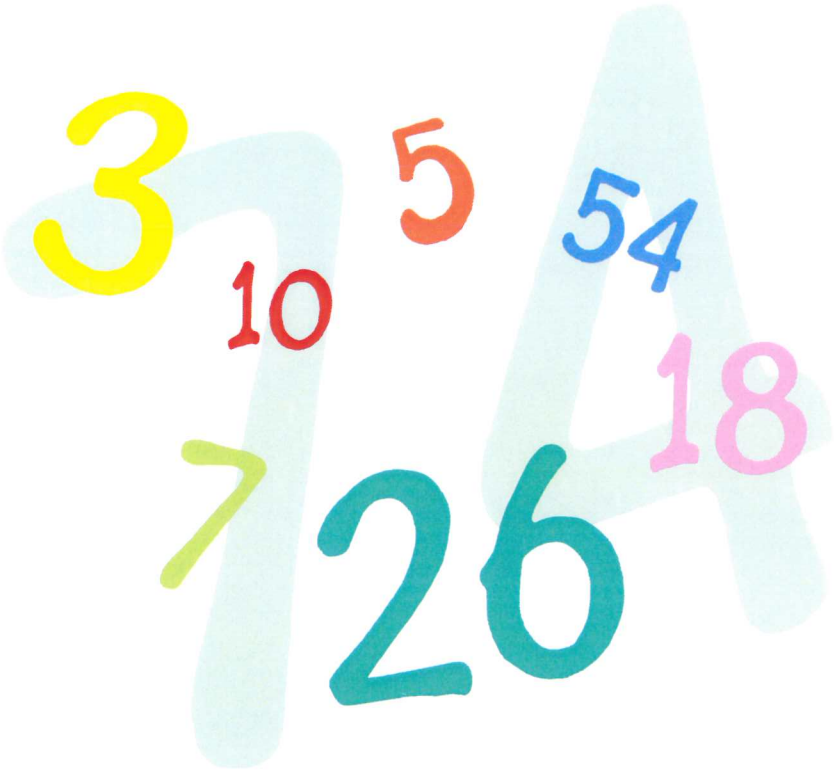
3) Employees receive a 4% pay rise. After the pay rise Lauren receives £21,840. What was her pay before the increase?

4) The population of birds on an island decreased by 2.5% in one year to 6435. What was the population before the decrease?

5) Carrie invests some money at a rate of 8% per annum. After 1 year there is £2754 in the account. How much did she invest?

Maths

Higher Tier

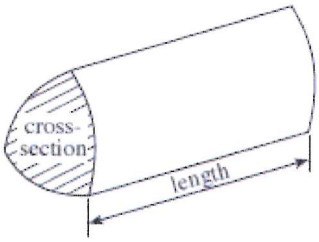


Number Revision

Percentages

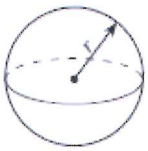
Formula List

Volume of prism = area of cross-section × length



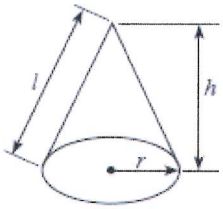
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

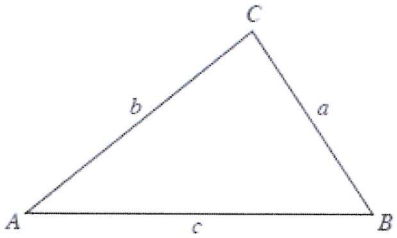


In any triangle ABC

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n$, having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

(185-04)

Percentages

'Percent' means 'out of 100'. If 90 per cent of the population owns a mobile phone, this means 90 out of every 100 people have one. The symbol '%' means per cent.

Key Point: A percent is just a fraction whose denominator (bottom) is 100.

So, if we say "52%", what we mean is $\frac{52}{100}$.

Remember: Take care when asked to find the original price of something.

E.g. A TV set costs £190 in the sale.

What did it cost **before** the sale?

The answer is **NOT** £199.50!

Skills you need: To change back and forward from percentages to fractions and decimals

Simple and Compound Interest

Percent Change

Percentages of Amounts

Revision: MyMaths – Number – Percentages

1. Henry places £6000 in an account whic pays 4.6% compound interest each each.

Calculate the amount he has in his account after 2 years.

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2. Sarah puts £8600 in a bank. The bank pays compound interest of 3.8% per year.

Caluclate the amount Sarah has in her account after 4 years.

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3. Mary deposits £10000 in an account which pays 5.6% compound interest per year.

How much will Mary have in her account after 5 years?

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4. Laura buys a new car for £14600.

The annual rate of depreciation 23%.

How much is the car worth after 3 years?

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5. Dave places £17000 in an account whic pays 4% compound interest per year.

How many years will it take before he has £19122.68 in the bank?

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6. A bike is sold for £199.99 including 17.5% VAT.

What was the priceof the bike before the VAT was added?

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7. Becky's wage increases from £5.31 per hour to £6.05 per hour.

What percent increase is this?

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8. Mick's weekly wage is now £343.75 following a 25% increase.

What was his wage before the increase?

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9. 65% of the members of a sports club are male.

If there are 245 female members how many members are there altogether.

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10. Four friends share a meal which costs £26.50. They decided to leave a 20% tip.

If the total is shared equally how much will each friend pay?

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11. A coat priced at £57.00 is reduced to £51 in a sale.

What percent reduction is this?

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12. Ashley scored 31 out of 40 in a test.

What percent did he get?

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13. Increase £420 by 32%

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14. In a survey 38% of people picked football as their favourite sport.

How many degrees would you need to shoe this on a pir chart?

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15. The population of Gemtown at the end of 2006 was 354.

If the population had incresed by 18% during the year what was the population at the end of 2005?

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16. A market trader buys 25kg of apples at 40p per kg. She makes 70% profit on the first 20kg of apples she sells. The rest she sells for 80% of the price she paid fot them.

How much profit did she make?

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Chapter 2 – Processing and Representing Data

Tables

Databases – Tables with a collection of data. They are a form of secondary data is the data is available online and, in most cases, easily accessible.

These tables usually contain information from real-life statistics, and you will be asked in the exam to extract and interpret information from it. These questions have multiple parts and many 1 marker sub-questions. You need to be able to use these tables to identify values, calculate totals/differences/percentages, describe trends and explain inconsistencies. One of the main inconsistencies will be that the percentages do not add up to 100% and this is due to rounding errors because individual percentages for columns/rows in the tables have been rounded.

As the data represents real-world statistics you may be asked to explain reasons for trends. Think about the data in terms of real-life rather than just an exam question. What real-life situation may affect the data you have?

Make	September 2016		September 2017		% change in sales
	sales	market share (%)	sales	market share (%)	
Ford	49 078	10.45	39 696	9.31	−19.12
Volkswagen	33 722	7.18	36 332	8.53	7.74
BMW	32 595	6.94	31 465	7.38	−3.47
Mercedes-Benz	31 988	6.81	31 430	7.37	1.74
Vauxhall	41 697	8.88	31 058	7.29	−25.52
Audi	31 113	6.62	29 619	6.95	−4.80
Nissan	27 807	5.92	28 810	6.76	3.61
Toyota	18 888	4.02	19 222	4.51	1.77
Hyundai	17 039	3.63	16 587	3.89	−2.65
Kia	15 340	3.27	15 706	3.69	2.39
Land Rover	14 629	3.11	14 504	3.40	−0.85
Peugeot	16 130	3.43	12 810	3.01	−20.58
Renault	17 275	3.68	12 378	2.90	−28.35
Mini	13 119	2.79	12 282	2.88	−6.38

(Source: www.smmt.co.uk)

Two-Way Tables – Has information in two categories and has two variables so the data is called bivariate data.

To find missing values, start with the row or column that has only one value missing. Make sure the grand totals for the rows and columns add up to the same number.

When comparing data from two-way tables, write about comparisons between rows/columns but also individual cells.

Age	male	female	Total
18 to 22	2	4	
23 to 29	15		
30 to 36			21
Total	30	30	

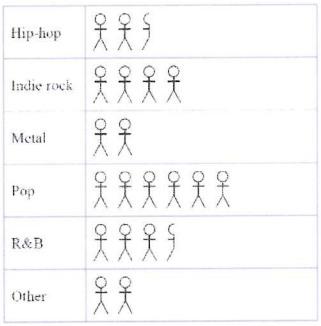
(Source: www.wtatennis.com and www.atpworldtour.com)

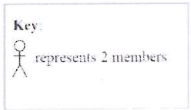
Pictograms

Uses pictures or symbols to represent a particular amount of data. Always has a key to show the amount each symbol represents.

When drawing a pictogram, make sure that:

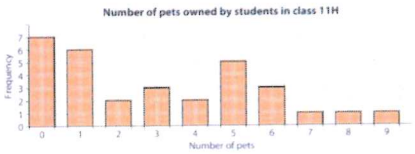
- Each symbol is the same size
- The symbols represent numbers that can be easily divided to show different frequencies, e.g. for a symbol that represents 4, you can draw a quarter of the symbol to show a frequency of 1.
- Spacings are the same in each row.
- There is a key to show the frequency that each symbol represents.





Bar Charts

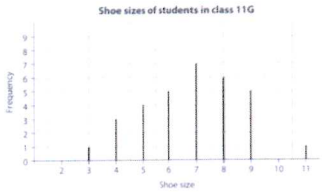
- **Simple Bar Charts**



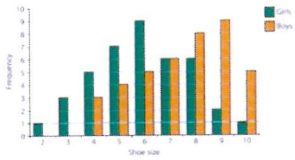
  - Bars are equal width
  - Equal gaps between bars
  - Frequency on y-axis

- **Vertical Line Graph**

Similar to simple bar chart but with lines instead of bars.



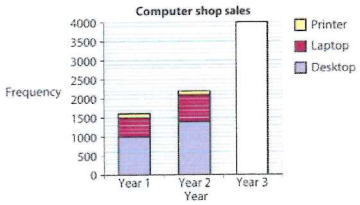
- **Multiple Bar Charts**



Can be used to compare two or more sets of data.
Has more than one bar for each class represented by different colours which is shown in the key.

- **Composite Bar Charts**

Has single bars split into different sections for each different category. Usually used to compare different times/days/years. The frequency of each component should be calculated by subtracting the upper frequency of that component with the lower frequency. Do not just read off the y-axis (unless looking at total frequencies or the bottom component).



Stem and Leaf Diagrams

A good way of organising data without losing any of the detail – All the original data is in the diagram but looks simple. It also shows the shape of the distribution – whether most of the data lies at the beginning, the end or is distributed in the middle.
Each value is split into a 'stem' and 'leaf' – Stems can be more than one digit, leaves are single digits only. No need for commas in between leaves. Leaves must be written in order from smallest to largest – this makes it easier to find mode and median.

How to draw one:

1) Put the **first digits** of each piece of data in **numerical order** down the left hand side.
2) Go through each piece of data in turn and put the remaining digits in the **correct row.**
3) Re-draw the diagram, putting the pieces of data in **numerical order**.
4) Add a **key**.

- **Back-to-back Stem and Leaf Diagrams**
  - Shows two sets of data sharing the same stem so that you can easily compare them.
  - Numbers closest to the stem are smallest.
  - Use two different keys for each set of data.

Pie Charts

A way of displaying data to show how something is shared or divided into categories,
Each sector shows what proportion that category represents of the total data,
Area of Pie Chart = Total Frequency
Angles add up to 360^0.

How to draw a pie chart:

1. Total up the frequency
2. Calculate the angle for each frequency. 360/frequency
3. Calculate the angle for each category will be by multiplying your previous answer by the frequency.
4. Make sure all the angles now add up to 360.
5. Draw the pie chart.
6. Label the sectors.

Interpreting Pie Charts – Remember pie charts show proportion and not numbers.

Comparative Pie Charts

Can be used to compare two sets of data of different sizes. The areas of the two circles should be in the same ratio as the two frequencies.
Why? Drawing two pie charts the same size can be misleading.
Area of Pie Chart = Total Frequency
So, the larger the pie chart, the greater the frequency.
To compare the total frequencies, compare the areas.

Working out radius of second pie chart:

1. Divide both areas (this gives you the area scale factor)
2. Square root answer (this gives you the scale factor for radius)
3. Multiply by radius of first pie chart.

$$r_2 = r_1 \frac{\sqrt{F_2}}{\sqrt{F_1}}$$

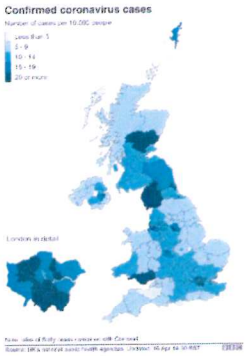
- If pie chart B is larger than pie chart A then pie chart B has a greater frequency.
- If both pie charts then have the same angle for a sector that means that sector has a greater frequency in pie chart B even though the proportions are the same because it has a larger area.

Population Pyramids

Shows distribution of ages in a population, in numbers or proportion/percentages.
They are used to compare two sets of data, usually genders or two geographical areas.
When comparing the data look at the shape of the distribution.

- If it looks like a pyramid with smaller bars at the top that means there is a higher proportion of younger people in the population and less older people. This could be because if short life expectancy (how long people live), high birth rates or high death rates.
- If the diagram looks more or less straight that means there is a similar proportion of older and younger people in the population which could be because of lower birth/death rates or that the life expectancy is increasing.
- An upside-down pyramid with larger bars at the top and smaller bars at the bottom shows that the population has a larger proportion of older people compared to younger people. This could be because of low birth/death rates, longer life expectancy or the location might be far from the city or a coastal area where older people are retiring to.

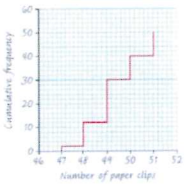
Choropleth Maps (not Chloropeth)



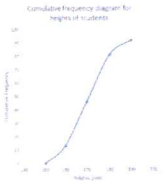
Think colour by numbers.
They split a geographical area into different regions which are then shaded.
The darker the shading the higher the frequency for that area.
Each map has a key to show what the shading represents.
Interpreting:
The area of the map which is shaded darkest has the highest proportion/percentage.
Look at the key for the shading to read off percentages/numbers.

Cumulative Frequency Diagrams

Cumulative frequency is a running total of the frequencies.
To work out CF for a class interval, add all the frequency for that class interval and the CF of the previous class interval.
Use upper bounds for x-axis when plotting points.



CF Step Polygons – Use for **discrete** data.
Plot the points using upper bound of class interval and join points using straight lines by going *across then up*.



CF Curves – Use for **grouped continuous** data.
Plot points using upper bound of class interval and connect with a *smooth curve.*

Estimating values from CF diagrams:

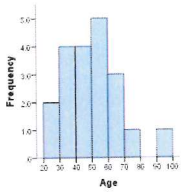
- Median
  - Work out median value by dividing total frequency by 2.
  - Find on Y-axis
  - Draw horizontal line from that value to curve/line
  - Read off value from x-axis
- Interquartile Range (IQR)
  - Work out 25% and 75% values
  - Find on y-axis
  - Draw horizontal line from that value to curve/line
  - Read off values from x-axis
  - Subtract them (Big one – small one)
- Estimating more than/greater than values
  - Draw a vertical line from the value in the question on the axis to the curve.
  - Read off corresponding y-axis value.
  - Subtract from total frequency.

Histograms

Represents continuous data from grouped frequency tables.
No gaps between bars.

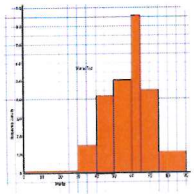
Equal Class Widths

x-axis = data
y-axis = frequency
Looks like bar charts without gaps.



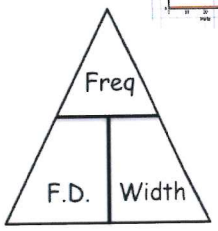
Unequal Class Widths

Area of bar = frequency
Y-axis = Frequency Density (not frequency)
The idea is that the frequency density reflects the 'concentration' of things within each range of values.



$$Frequency\ Density = \frac{Frequency}{Class\ Width}$$

$$Frequency\ Density \times Class\ Width = Frequency$$



Drawing Histograms:

1. Calculate class widths for each class interval
2. Calculate frequency density for each class interval using FD = F/CW formula.
3. Draw a suitable scale on y-axis labelled frequency density.
4. Draw bars using frequency density data. (Remember the bars have no gaps in between)

Estimating frequencies from histograms:

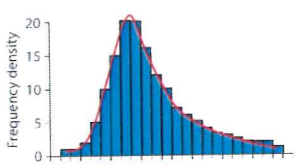
With these questions you are using the class widths and frequency density from the histogram to work out frequencies. Be careful when calculating class width as some intervals may not include the entire bar.

1. Find the bars that cover the range you need from the question.
2. Work out the frequency for each bar using the FD X CW = F formula.
3. Add the frequencies.

To compare histograms, they need to have the same class intervals and frequency density scales.
When comparing histograms, describe the shape of the distribution and what this shows.

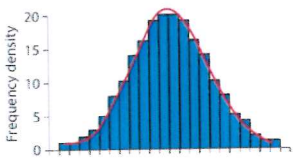
The Shape of a Distribution

This is the shape formed by the diagram. It can be positive, negative or symmetrical.

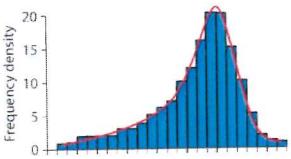


This distribution has positive skew. Most of the data values are at the lower end. Example: The age at which a person learns to write.

The distribution is stretched out in the positive direction →.



This distribution is symmetrical. It has no skew. Example: The lengths of leaves on a tree.



This distribution has negative skew. Most of the data values are at the upper end. Example: The age at which a person dies.

The distribution is stretched out in the negative direction ←.

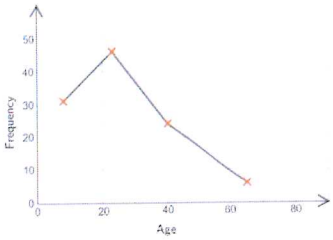
Frequency Polygons

Similar to histograms with equal class widths but without the bars.

Uses mid-points of class intervals and points are plotted and then joined together with straight lines.

Common errors:

- Midpoints not used
- Joined together at the bottom
- Points not joined together with straight lines but with a curve instead.



Misleading Diagrams

Diagrams can be misleading because of their shape of because of axes and scales.

Types of Misleading Diagrams:

- Pictograms – Same symbol and size needs to be used for all the diagrams and key needed.
- 3D charts – They distort parts of the diagram making it difficult to read off values.
- Colours – Some colours may make parts of the diagram stand out more thus making it seem more important when it may not be.
- Lines that are drawn too thick making it difficult to read information from the diagram.

Axes and Scales that can be misleading:

- Scales that do not start at zero.
- Missing values on the scales.
- Axes that are unevenly scaled.
- Axes that are not labelled.
- Not using a key.

Stem and Leaf Diagrams

Below are the heights in metres of a sample of people.

1.69	1.39	1.39	1.42	1.33	1.6	1.33	1.26	1.22	1.28
1.62	1.78	1.24	1.64	1.32	1.78	1.24	1.57	1.42	1.38
1.73	1.68	1.25	1.24	1.74	1.37	1.55	1.2	1.85	1.78
1.71	1.22	1.83	1.86	1.25	1.74	1.89	1.34	1.88	1.38
1.27	1.85	1.63	1.81	1.25	1.87	1.69	1.7	1.51	1.21

Using your knowledge of stem and leaf create a suitable stem and leaf diagram.

Part of the diagram has been started for you. You need to decide upon a suitable stem and then complete the leaves. From this find the median and the range. Remember to include a key when you draw your diagram

Part One (Unsorted)

Part One (Sorted)

Key: .....|..... represents .....

Range = ...

Median = ...

Below are the heights in metres of a sample of people.

1.69	1.39	1.39	1.42	1.33	1.6	1.33	1.26	1.22	1.28
1.62	1.78	1.24	1.64	1.32	1.78	1.24	1.57	1.42	1.38
1.73	1.68	1.25	1.24	1.74	1.37	1.55	1.2	1.85	1.78
1.71	1.22	1.83	1.86	1.25	1.74	1.89	1.34	1.88	1.38
1.27	1.85	1.63	1.81	1.25	1.87	1.69	1.7	1.51	1.21

Using your knowledge of stem and leaf create a suitable stem and leaf diagram.

Part of the diagram has been started for you. You need to decide upon a suitable stem and then complete the leaves. From this find the median and the range. <u>Remember to include a key when you draw your diagram</u>

Part One (Unsorted)

1.2	7	2	4	5	4	5	5	4	6	0	2	8	1
1.3	9	9	3	2	7	3	4	8	8				
1.4	2	2											
1.5	5	7	1										
1.6	9	2	8	3	4	0	9						
1.7	3	1	8	4	8	4	0	8					
1.8	5	3	6	1	7	9	5	8					

Part One (Sorted)

1.2	0	1	2	2	4	4	4	5	5	5	6	7	8
1.3	2	3	3	4	7	8	8	9	9				
1.4	2	2											
1.5	1	5	7										
1.6	0	2	3	4	8	9	9						
1.7	0	1	3	4	4	8	8	8					
1.8	1	3	5	5	6	7	8	9					

Key: ...1.2|4..... represents ...1.24cm

Range = 1.89-1.2 = 0.69

Median = 1.53

Back-to-Back Stem and Leaf Diagrams 1

Question1: For each of the data sets, draw a back-to-back stem and leaf diagram, and from them find the median and range for the data.

The table shows Year 9 test results, out of 100

Boys	19	53	47	81	38	61	67	73	79	49	52	58	62	65
Girls	85	24	32	36	45	49	56	71	80	51	49	55	47	50

Key: |3|5 represents 35 3|2| represents 23

Girls (Unsorted)		Boys (Unsorted)
	0	
	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	

Girls (sorted)		Boys (sorted)
	0	
	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	

Median for girls =
Range for girls =

Median for boys =
Range for boys =

What do these results tell you? Remember to include mathematical terms.

.....

.....

.....

Back-to-Back Stem and Leaf Diagrams 1

Question1: For each of the data sets, draw a back-to-back stem and leaf diagram, and from them find the median and range for the data.

The table shows Year 9 test results, out of 100

Boys	19	53	47	81	38	61	67	73	79	49	52	58	62	65
Girls	85	24	32	36	45	49	56	71	80	51	49	55	47	50

Key: |3|5 represents 35 3|2| represents 23

Girls (sorted)												Boys (sorted)										
											0											
											1	9										
										4	2											
									6	2	3	8										
							9	9	7	5	4	7	9									
							6	5	1	0	5	2	3	8								
											6	1	2	5	7							
										1	7	3	9									
									5	0	8	1										
											9											

Median for girls = 49.5
Range for girls = 85-24 = 61

Median for boys = 59.5
Range for boys = 81-19 = 62

What do these results tell you? Remember to include mathematical terms.

The boys did better than the girls using the median as a comparison. The boys and girls range is similar

Data Analysis Recap

Problem 1

Debbie is carrying out a survey to see how much people spend on groceries. She has the following information about which people shop at which supermarket in her town.

Supermarket	Sainsbury's	Tesco	Aldi	Waitrose	M&S
No of shoppers	6400	5100	4500	2000	1350

She wants to conduct a stratified sample of 200 people, how many people should she survey from each supermarket?

Sainsbury's

Tesco

Aldi

Waitrose

M&S

Problem 2

Use the grouped frequency table to generate a cumulative frequency table for the time it takes 16 people to travel to university.

Time (t) in minutes	Frequency
$10 < w \leq 20$	6
$20 < w \leq 30$	7
$30 < w \leq 40$	3

Time (t) in minutes	Cumulative Frequency
$10 < t \leq 20$	
$10 < t \leq 30$	
$10 < t \leq 40$	

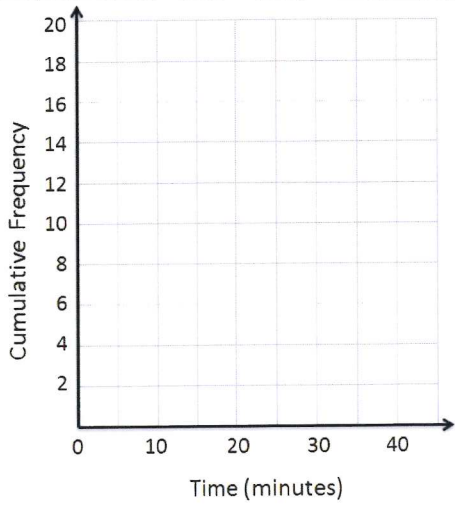
Problem 3

a) Use your cumulative frequency table above to draw a cumulative frequency graph.

b) Use your cumulative frequency graph to estimate the interquartile range and median.

Median: _____ minutes

IQR: _____ minutes



Problem 4

Each student at a school has a student identification number. Counsellors have a computer generate 505050 random identification numbers and those students are asked to take a survey. **What type of sampling is this?**

a) Simple random sampling

b) Stratified random sampling

c) Cluster random sampling

Problem 5

Find the Lower Quartile, Median and Upper Quartile of the following heights of Year 12 boys.

1.34m 1.38m 1.67m 1.68m 1.70m 1.87m 1.38m 1.50m

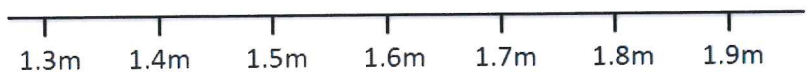
LQ = _____

Median = _____

UQ = _____

Problem 6

Draw a box plot to display the information in Problem 5.



Draw your box plot on the left.

Men's Ages		Women's Ages

Problem 7

Here are the ages of 11 men and 11 women in a tennis club:

Men:

22 31 37 23 17 28 19 25 31 32 26

Women:

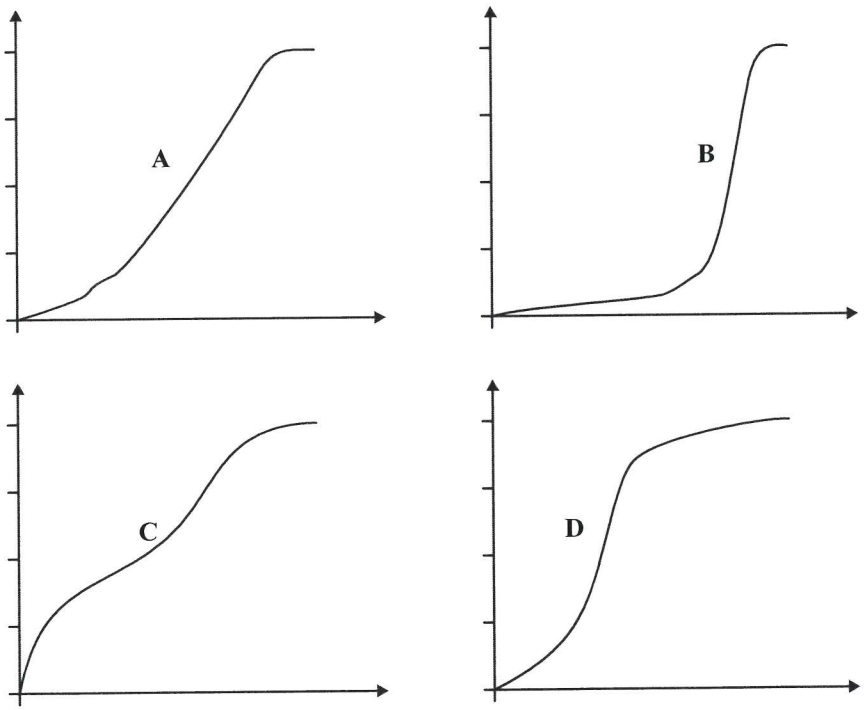
17 30 13 25 41 44 38 41 39 41 43

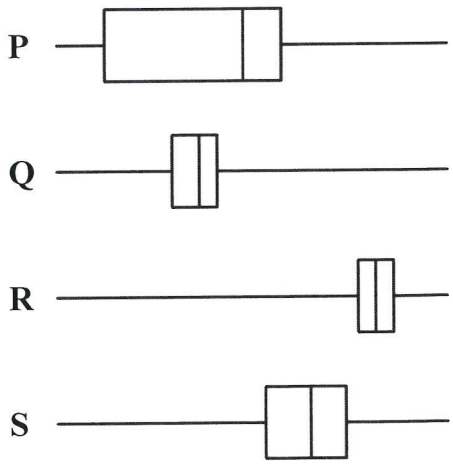
Complete the following back-to-back stem and leaf diagram.

Compare the ages of men and women at the tennis club

Problem 8

Here are four cumulative frequency diagrams. Here are four box plots. Match the box plot to the graphs.





Data Analysis Recap 2

Problem 1

The table gives information about the numbers of students in the two years of a college course.

Anna wants to interview some of these students.

She takes a random sample of 70 students stratified by year and by gender.

Work out the number of students in the sample who are male and in the first year.

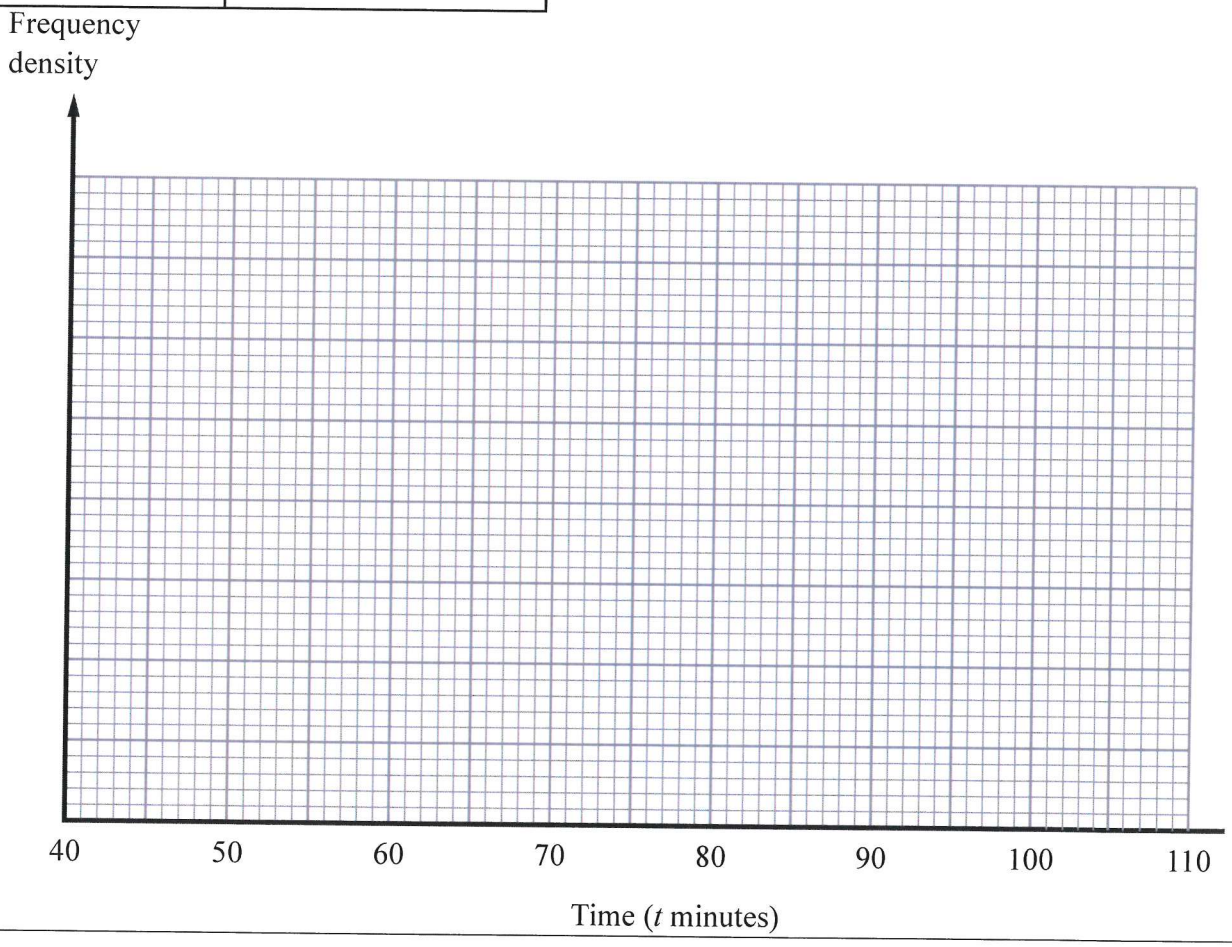
	Male	Female
First year	399	602
Second year	252	198

Problem 2

The table gives some information about the lengths of time some boys took to run a race.

Draw a histogram for the information in the table.

Time (t minutes)	Frequency
$40 \le t < 50$	16
$50 \le t < 55$	18
$55 \le t < 65$	32
$65 \le t < 80$	30
$80 \le t < 100$	24



Problem 3

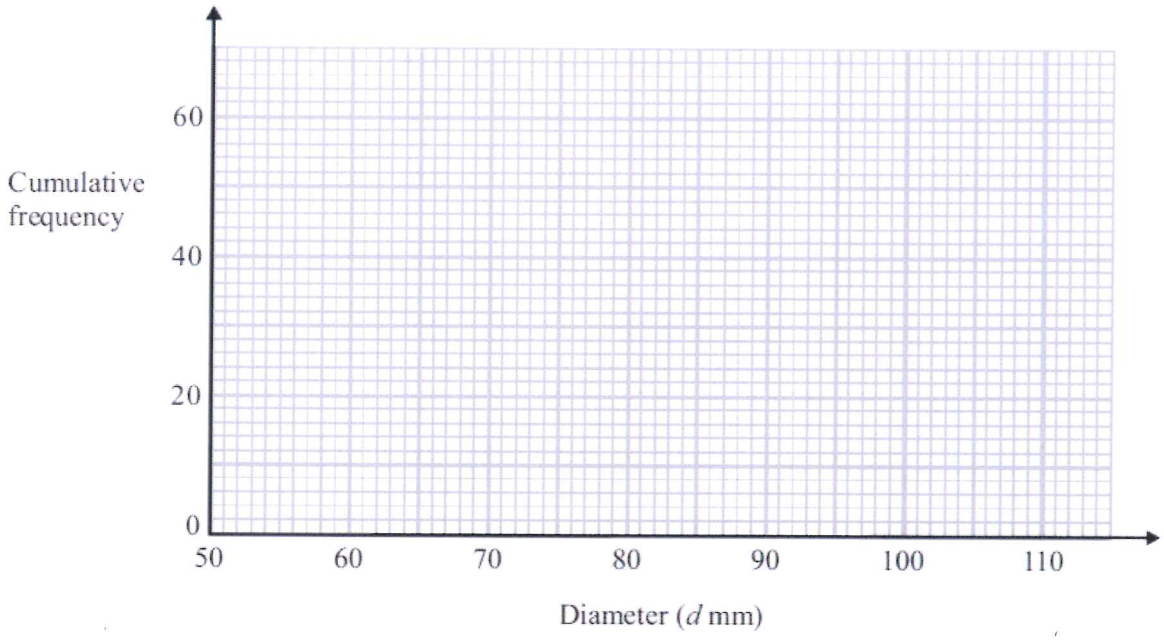
The cumulative frequency table shows information about the diameters of 60 oranges.

Diameter (d mm)	Cumulative frequency
$50 < d \leqslant 60$	12
$50 < d \leqslant 70$	42
$50 < d \leqslant 80$	54
$50 < d \leqslant 90$	57
$50 < d \leqslant 100$	59
$50 < d \leqslant 110$	60

(a) On the grid, draw a cumulative frequency graph for the table.

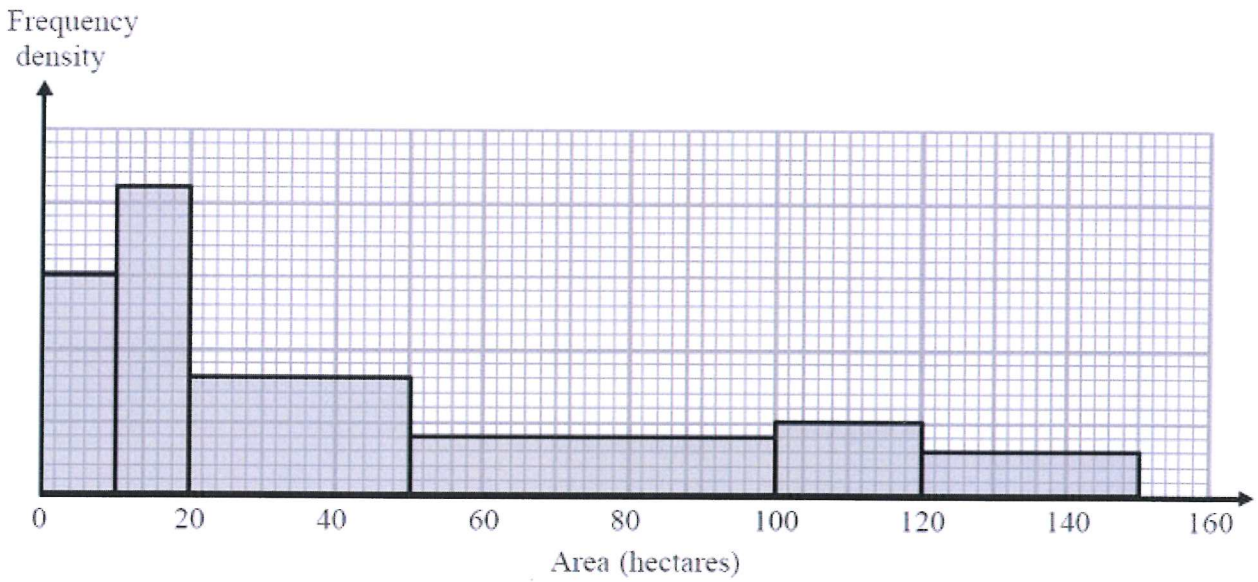
(b) Work out the IQR

(c) Estimate % of oranges that had diameter over 95mm?



Problem 4

The histogram shows information about the areas of some farms.



90 of the farms have an area of 10 hectares or less.

60% of the farms with an area of 100 hectares or less are arable farms.

$\frac{1}{2}$ of the farms with an area of more than 100 hectares are arable farms.

Work out an estimate for the total number of arable farms.

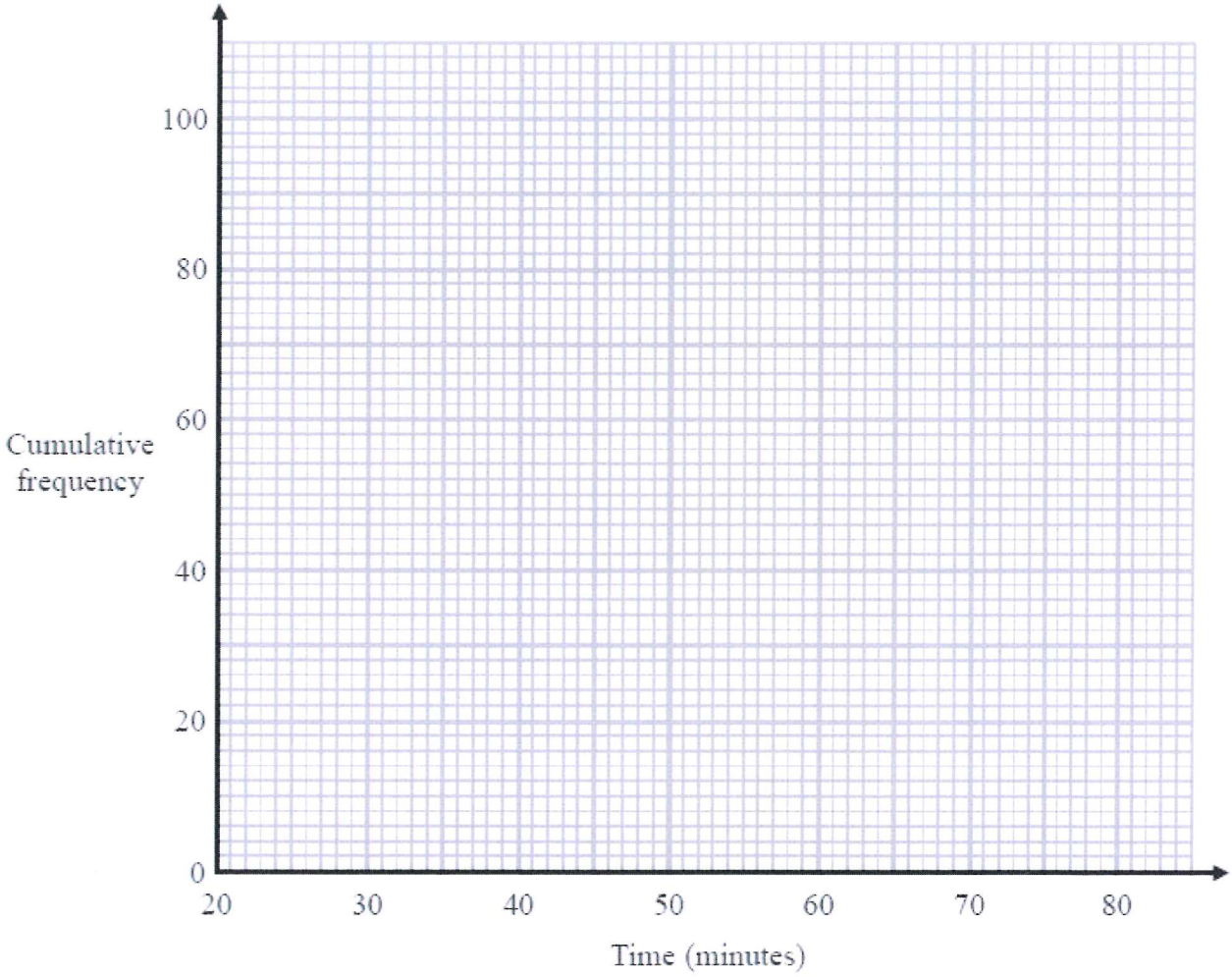
Core maths data analysis recap 3

Q1.

The table shows information about the times taken by 100 people in a fun run.

Time (t minutes)	Frequency
$20 < t \leqslant 30$	4
$30 < t \leqslant 40$	16
$40 < t \leqslant 50$	36
$50 < t \leqslant 60$	24
$60 < t \leqslant 70$	14
$70 < t \leqslant 80$	6

a) On the grid, draw a cumulative frequency graph for your table.

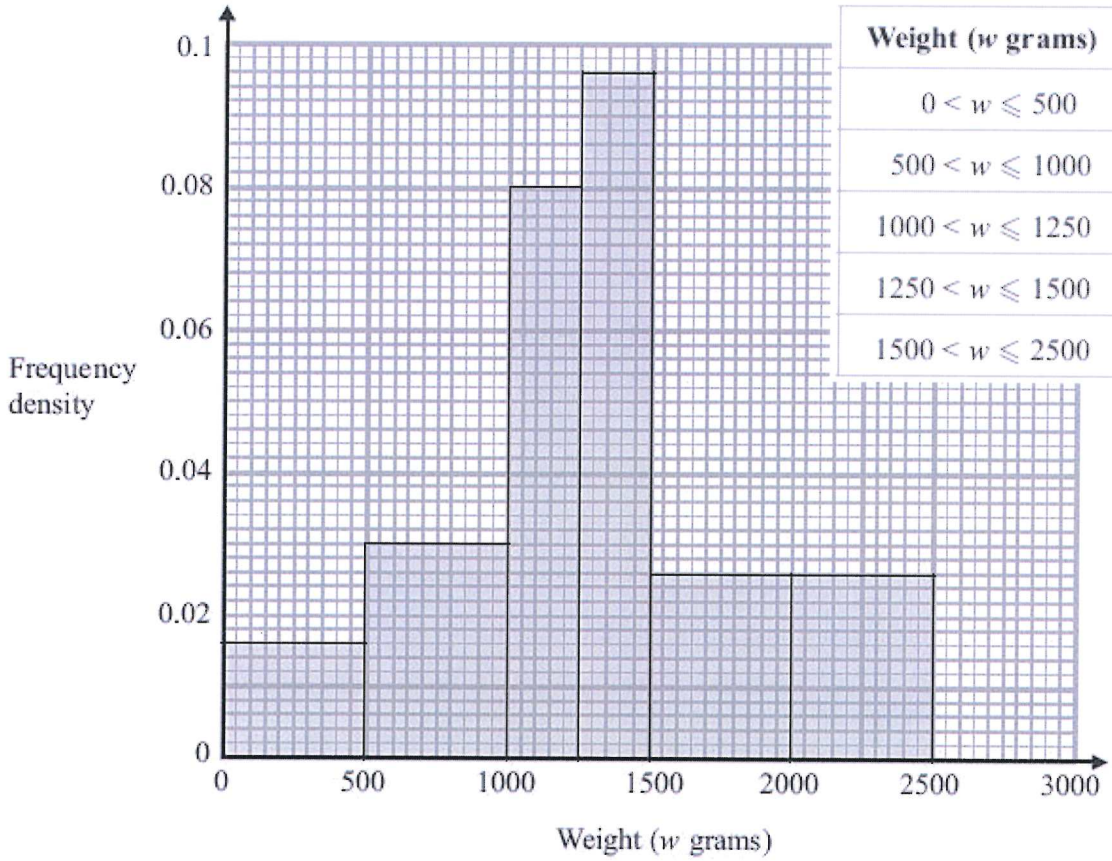


(b) Use your graph to find an estimate for the median time..... minutes

(c) Use your graph to find an estimate for the number of people who took longer than 63 minutes.....

d) Estimate the mean

2. Jim went on a fishing holiday.
The histogram shows some information about the weights of the fish he caught.



Weight (w grams)	Frequency
$0 < w \leqslant 500$	8
$500 < w \leqslant 1000$	
$1000 < w \leqslant 1250$	
$1250 < w \leqslant 1500$	
$1500 < w \leqslant 2500$	

(a) Use the histogram to complete the frequency table.

Jim kept all the fish he caught with a weight greater than 2000 g.

(b) Find the ratio of the number of fish Jim kept to the total number of fish he caught.

Q3.

There are 15 bags of apples on a market stall. The mean number of apples in each bag is 9
The table below shows the numbers of apples in **14** of the bags.

Number of apples	Frequency
7	2
8	3
9	3
10	4
11	2

Calculate the number of apples in the 15th bag.

Q4. Give one advantage and one disadvantage of using a random sample of 50 teachers in this school to find out there opinion Black Friday sales.

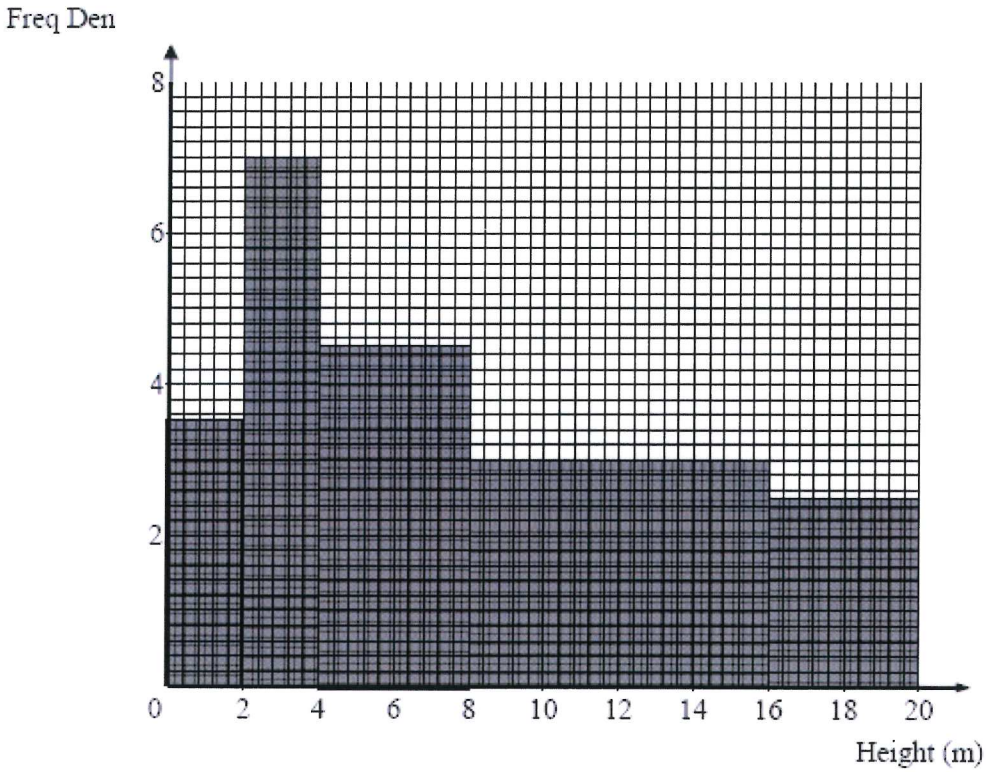
Mark Scheme

Q1.

PAPER: 1MA0_1H				
Question	Working	Answer	Mark	Notes
(a)		4, 20, 56, 80, 94, 100	1	B1 cao
(b)		graph	2	M1 ft from their table for at least 5 points plotted correctly at the ends of the intervals provided table values are cumulative, condoning one arithmetic error, or if the shape of the graph is correct for 5 or 6 points plotted not at the ends but consistently within each interval and joined A1 cao for correct graph with points joined by curve or straight line segments
(c)		47 to 49	1	B1 for 47 to 49 or ft their cf graph at cf = 50
(d)		13 to 16	2	M1 for reading a value from their cf graph at time = 63 (84 to 87) A1 for answer in the range 13 to 16 or ft from their graph

2)

	Working	Answer	Mark	Notes
(a)	0.016 × 500 = 8 0.03 × 500 = 15 0.08 × 250 = 20 0.096 × 250 = 24 0.026 × 1000 = 26	(8), 15, 20, 24, 26	2	M1 for correct calculation to find one frequency e.g. 0.03 × 500 or 0.08 × 250 or 0.096 × 250 or 0.026 × 1000 or for one frequency correct or establishing that 1 cm^2 = 2.5 fish A1 for all frequencies correct
(b)	0.026 × 500 = 13 8 + 15 + 20 + 24 + 26 = 93	13 : 93	2	M1 ft for a complete correct method to find the number of fish over 2000 g ie 0.026 × 500 (=13) or '26'÷2 A1ft for 13 : 93 or '0.026 × 500' : 'total for all their fish' or '26'÷2 : 'total for all their fish'



Q3.

Question	Working	Answer	Mark	Notes
	15 × 9 = 135 (7 × 2) + (8 × 3) + (9 × 3) + (10 × 4) + (11 × 2) = 127 135 - 127	8	3	M1 for 15 × 9 or 135 seen M1 (7 × 2) + (8 × 3)+(9 × 3) + (10 × 4) + (11 × 2) or 127 seen A1 8 cao

Core maths data analysis recap 6

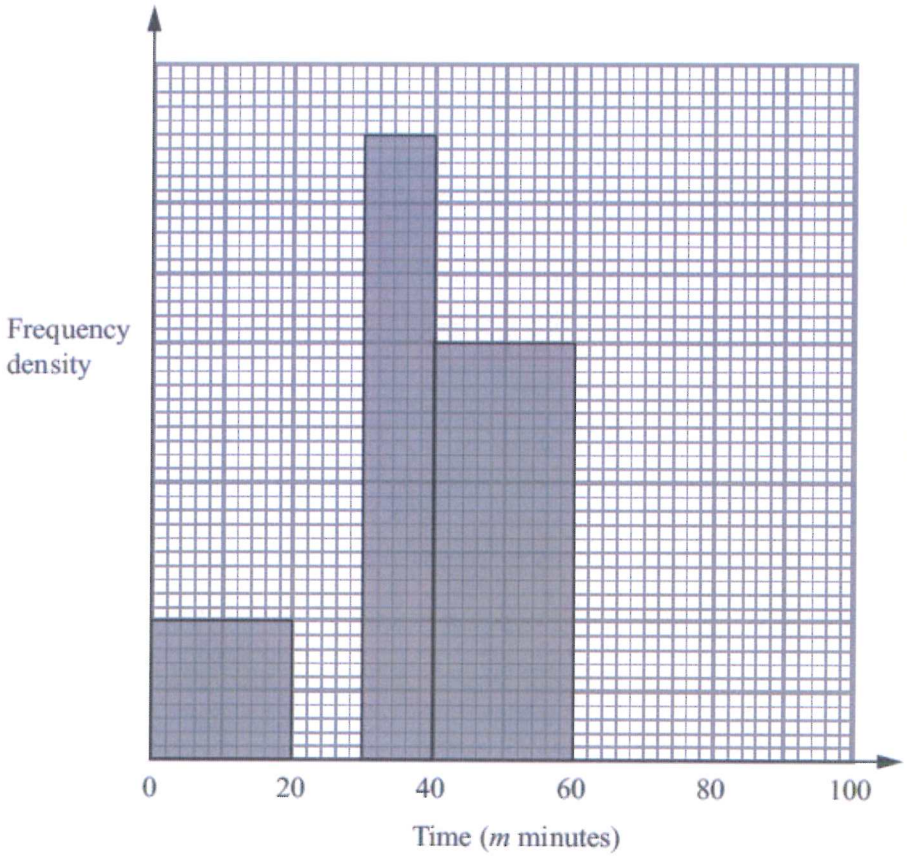
1. The table and the histogram show some information about the time, in minutes, taken by a group of students to travel to college in one week.

(a) Use the histogram to complete the table.

(b) Use the table to complete the histogram.

(c) Work out an estimate for the median time.

(d) Work out the upper and lower quartile



Time (m minutes)	Frequency
$0 < m \leqslant 20$	20
$20 < m \leqslant 30$	30
$30 < m \leqslant 40$	
$40 < m \leqslant 60$	
$60 < m \leqslant 100$	48

2.

Beth is a GP on a salary of £86 000 per year; she is employed on a 0.75 full-time equivalent contract.

(a) How much does she earn each year, before deductions?

(b) How much are her total income tax and NI contributions this year?

3. Last week Theresa May survived a vote of no-confidence triggered by members of her own party over her handling of Brexit.

The final vote:

The Conservative Party members voted 200 vs. 117 in favour of May staying as prime ministers.

The BBC are going to interview 20 of The Conservative Party members on their opinion of what they think should happen next.

a) Explain why taking a random sample of The Conservative Party members would not be the most suitable sampling method.

b) The BBC decides to use a stratified sample.

i) What % of the 20 people interviewed should be members who voted for May to stay?

ii) How many people should be interviewed from the group of members that voted for her to go? (Not be Prime Minister any more)

4. Sara works part-time, for 15 hours per week. She is paid £12.50 per hour.

(a) Does she pay income tax? If 'Yes', how much per year?

(b) Does she make NI contributions? If 'Yes', how much are her contributions per year?

c) What is Sara's net pay for the year?

Answers

1) **Median 42.17 (Total students 203)**

Q1 = 30.17

Q3 = 59.1

2) **(a) £64,500**

(b) NI =£4914.12

Tax = £14160

Total = £19072.12

3)a) It would not necessarily be representative of the party as you could end up sampling all members in favour or not in favour.

b) 63%

c) 7 or 8

4) a) No

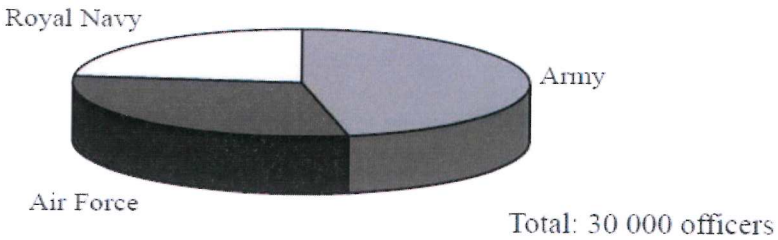
b) £159.12

c) £9590.88

Core maths data analysis recap 8

The pie chart shows information about the numbers of officers in the Royal Navy, Army and Air Force.

Officers in the Royal Navy, Army and Air Force



The pie chart is misleading.

(a) Write down one reason why.

.....

A new pie chart is to be drawn which is not misleading.

The number of officers in the Royal Navy is 7000

(b) Calculate the angle needed for the Royal Navy.

There are 85 girls and 65 boys in Year 12 of a school.

A sample of students is to be taken.

(a) The headteacher decides to choose a sample stratified by gender.

Is this a suitable sampling method to use?
Give reasons for your answer.

[2 marks]

(b) There will be 34 girls in the sample.

Describe how random numbers could be used to select a sample of the 34 girls.

[3 marks]

(c) For a different sample, the headteacher decides to choose all the girls in the two tutor groups closest to her office.

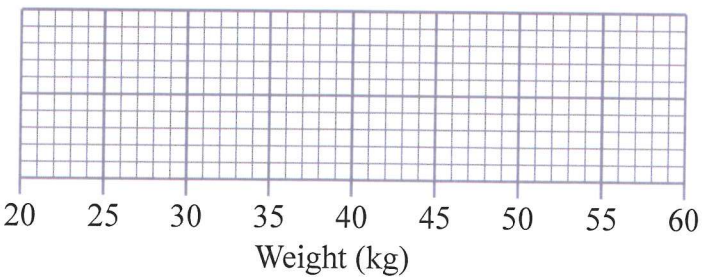
What type of sampling method is this?

[1 mark]

The weights, to the nearest kilogram, of 19 pigs were

36 38 30 31 38 43 55 38 37 30 48 41 33 25 34 43 37 40 36

Draw a box plot for this data.



(3)

In an experiment a biologist records the length, x cm, of 125 worms.

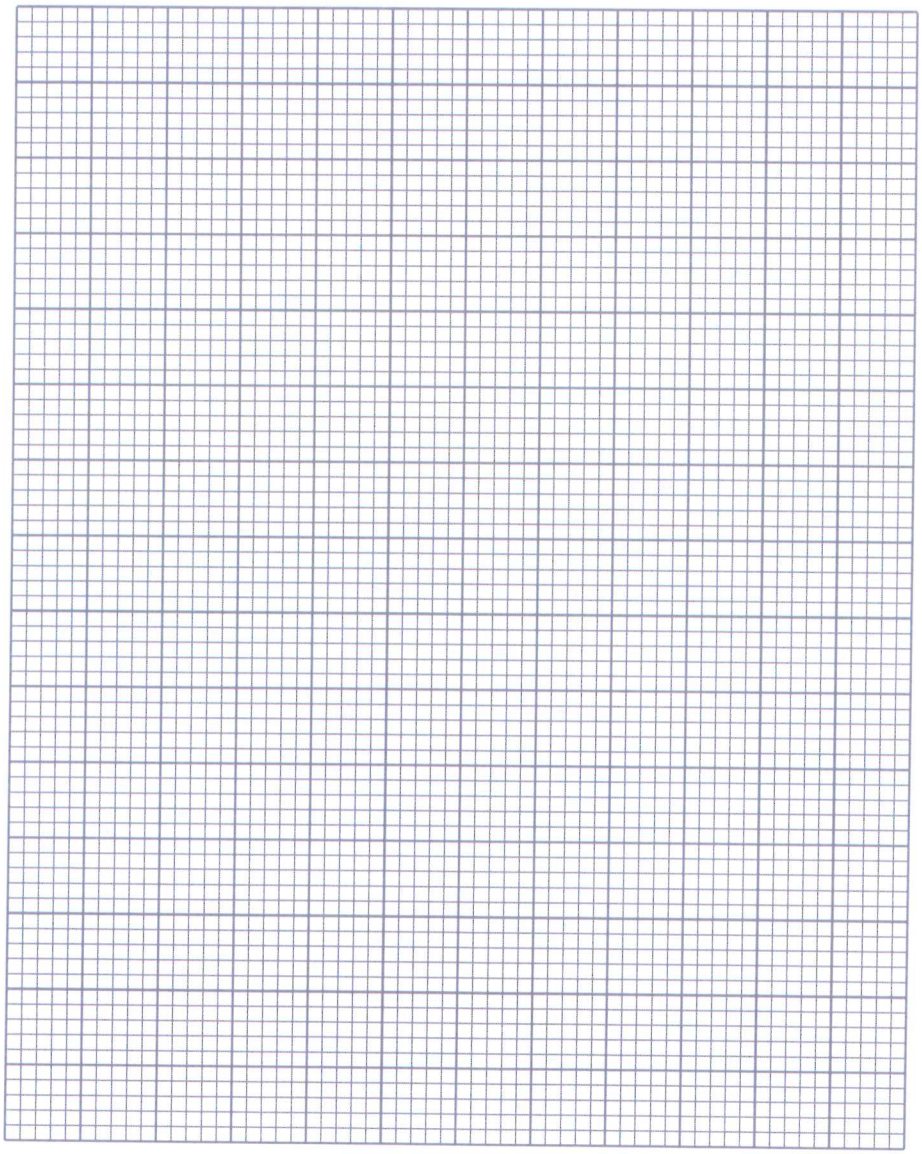
The results are summarised in the table.

Lengths in cm	Number of worms (f)
$0 < x \leq 4$	36
$4 < x \leq 6$	26
$6 < x \leq 8$	20
$8 < x \leq 10$	12
$10 < x \leq 15$	21
$15 < x \leq 20$	10

(a) Show that an estimate for the mean length of the worms is 7.1 cm.

(3)

(b) Draw a histogram to represent the data in the table.



(3)

(c) Estimate the median length of the worms from the histogram

(3)

Box and Whisker Diagrams (Box Plots)

Raw data (a long list of numbers) gives little insight into what is going on:

- The weights (KG) of 15 children: 37, 42, 31, 35, 48, 29, 50, 36, 44, 28, 63, 35, 41, 52, 43

We can calculate a few statistics to help sum up the data, but it's not very visual:

- Minimum = 28KG, Maximum = 63KG, Range = 35KG (how spread out is the data?)
- Mode = 35KG, Median = 41KG, Mean = 40.9KG (what is an average piece of data?)

A stem and leaf diagram gives a simple picture of the distribution of the data as well as the list of numbers – this can be really useful <u>for relatively small datasets</u>:

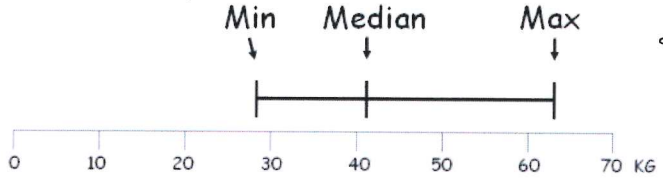
2	8 9
3	1 5 5 6 7
4	1 2 3 4 8
5	0 2
6	3

Key: 2 | 9 means 29

ORDERED STEM & LEAF

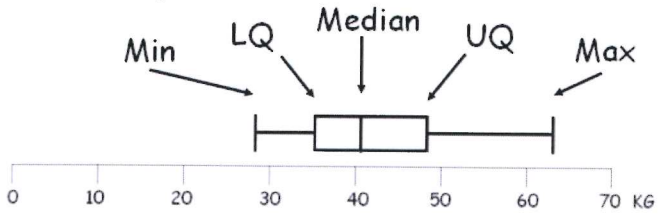
This contains all the raw data AND looks like a bar chart, so shows the spread of data! Very handy.

We could also plot our minimum, median and maximum like this:

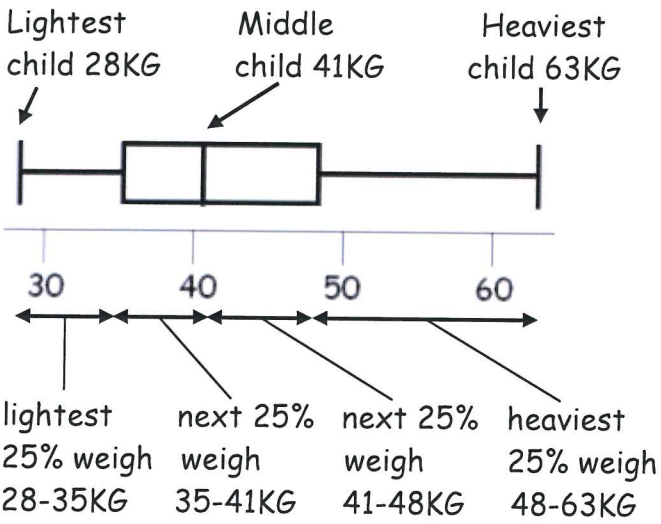


It looks pretty, but doesn't tell us very much apart from how spread-out the extremes are!

Then we could find the middle piece of data between the minimum and median (lower quartile) and the middle piece of data between the median and the maximum (upper quartile) and add them to the plot like this:



Interpreting a Box and Whisker Diagram



Calculating lower and upper quartiles

Lower Quartile

- $\frac{1}{4}(n + 1)^{th}$ piece of data (ordered)

  15 items of data ... n = 15

  $\frac{1}{4}(n + 1) = \frac{1}{4}(15 + 1) = 4^{th}$ item

  28, 29, 31, (35) 35, 36, 37, 41, 42, 43, 44, 48, 50, 52, 63

Upper Quartile

- $\frac{3}{4}(n + 1)^{th}$ piece of data (ordered)

  15 items of data ... n = 15

  $\frac{3}{4}(n + 1) = \frac{3}{4}(15 + 1) = 12^{th}$ item

  28, 29, 31, 35, 35, 36, 37, 41, 42, 43, 44, (48) 50, 52, 63

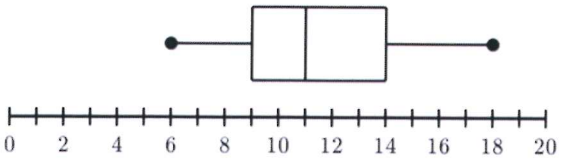
Sometimes questions use Q_0, Q_1 etc:
Q_0 = minimum, Q_1 = lower quartile,
Q_2 = median, Q_3 = upper quartile,
Q_4 = maximum

<u>Outliers:</u>

- Items of data smaller than LQ – 1.5xIQR might be considered 'outliers'
- Items of data larger than UQ + 1.5xIQR might be considered 'outliers'

Developing

Can you label this box plot with their values and what that represents?



How do you find the median?

How do you find the LQ and UQ?

Securing

Draw a box plot from the following data (make sure you draw one on top of another, so that they can be compared):

1) 3, 5, 6, 7, 7, 7, 7, 8, 9, 10, 12, 15, 16, 16, 16, 17
2) 2, 4, 4, 4, 5, 6, 6, 6, 9, 9, 9, 10, 10, 11, 20, 21

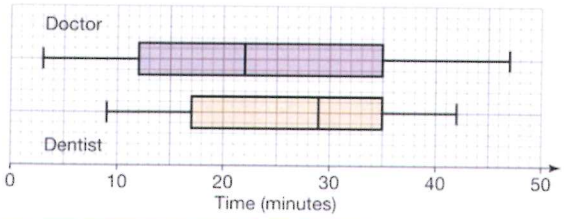
Mastering

What could the two box plots above represent?

Explain why the interquartile range may be a better measure of spread than the range.

How might you change this to make the range of each box plot the same?

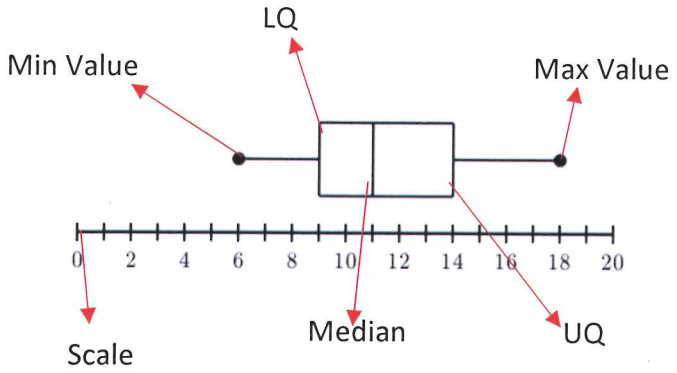
4 The box plots summarise the waiting times, to the nearest minute, of a group of patients at the doctor and the dentist.



Compare the box plots opposite, write two comparisons on them.

Developing

Can you label this box plot with their values and what that represents?



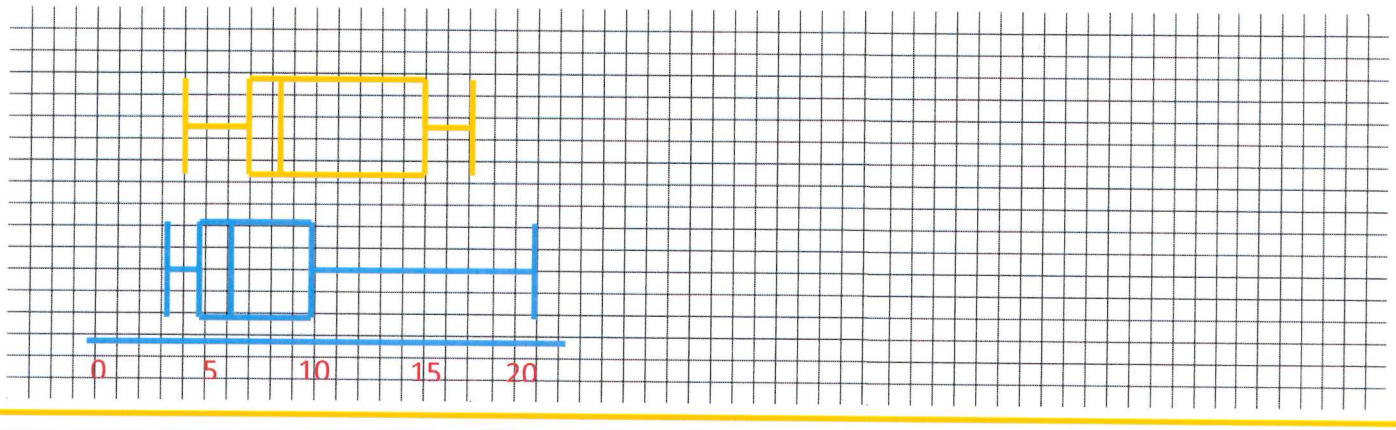
How do you find the median?
Put all of the values in order and then find the middle value. If there are two middle values, you find the halfway value between those two numbers.

How do you find the LQ and UQ?
The lower quartile is the halfway point between the lowest value and median. The upper quartile is halfway between the median and the biggest value.

Securing

Draw a box plot from the following data (make sure you draw one on top of another, so that they can be compared):

1) 3, 5, 6, 7, 7, 7, 7, 8, 9, 10, 12, 15, 16, 16, 16, 17
2) 2, 4, 4, 4, 5, 6, 6, 6, 9, 9, 9, 10, 10, 11, 20, 21



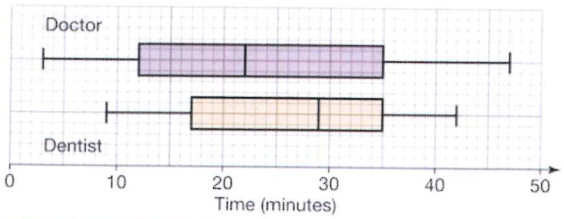
Mastering

What could the two box plots above represent?
Marks in a test, TV viewing figures etc. Anything that is measured with a number between 0 and 20.

Explain why the interquartile range may be a better measure of spread than the range.
The interquartile range is less affected by outliers as it doesn't use the first and last values which is used by the range.

How might you change this to make the range of each box plot the same?
You could – change the min or max values of both plots. You could change one box plot to match the other.

4 The box plots summarise the waiting times, to the nearest minute, of a group of patients at the doctor and the dentist.



Compare the box plots opposite, write two comparisons on them.
The range time to wait at the doctors is 44 minutes and 31 minutes at the dentist. There is a bigger spread of data for the doctors.

The longest wait by a patient at the doctors was 47 minutes and at the dentist 42 minutes.